HETEROGENEOUS TRAFFIC FLOW MODELLING
WITH THE LWR MODEL USING PASSENGER-CAR
EQUIVALENTS

Dr. Ir. Steven Logghe†,‡ and Prof. Ir. Ben Immers†,†

†Katholieke Universiteit Leuven
Department of Civil Engineering  - Transportation Planning and Highway Engineering
Kasteelpark Arenberg 40, 3001 Leuven (Belgium)
Phone: +32 16 32 16 69 / Fax: +32 16 32 19 76
E-mail: ben.immers@bwk.kuleuven.ac.be
http://www.kuleuven.ac.be/traffic

‡Transport & Mobility Leuven
Tervuursevest 54 bus 4, 3000 Leuven (Belgium)
Phone: +32 16 22 95 52 / Fax: +32 16 20 42 22
E-mail: steven.logghe@tmleuven.be
http://www.tmleuven.be

SUMMARY

This paper presents an extension of Lighthill, Whitham and Richards' dynamic traffic flow model (LWR) to a number of vehicle classes. Each class will be described by a separate fundamental diagram. When the various diagrams are similar in shape a scaling can be carried out that is related to the passenger-car equivalents. We present both an analytical framework and a numerical computation scheme that is based on the usual LWR-model scheme. An elaborated case illustrates the model.
INTRODUCTION

In the nineteen fifties Lighthill and Whitham (1) and Richards (2) independently developed the first dynamic traffic flow model. This LWR model describes the traffic on a link using a conservation law. It is, in addition, assumed that flow $q$ is related to density $k$. This equilibrium relation $Q_e(k)$ is better known as the fundamental diagram. The formulation below gives a partial differential equation for a freeway without access or exit ramps:

$$\frac{\partial k(x,t)}{\partial t} + \frac{d Q_e(k)}{dk} \frac{\partial k(x,t)}{\partial x} = 0$$  \hspace{1cm} (1)

The fundamental diagram $Q_e(k)$ was initially assumed to be smooth and concave, with a maximum flow $q_{max}$ for density $k_{max}$. The equilibrium flow equals zero at a 'jam' density $k_j$ and zero density. In that situation free-flow speed $u_f$ equals $Q_e'(0)$.

In the case of piece-wise initial and boundary conditions a meaningful solution to equation (1) can be derived analytically using characteristics, also called kinematic waves. Here characteristics are represented by straight lines with a slope of $Q_e'(k_0)$. These are drawn from a point with density $k_0$ in the $t-x$ plane. The traffic state at these solution lines equals the state at the boundary condition. Characteristics that travel against the traffic flow, in this case $Q_e'(k) < 0$ and $k > k_{max}$, correspond to congestion. Free-flowing traffic corresponds to characteristics with a positive slope so that $Q_e'(k) > 0$ and $k < k_{max}$.

When, as in figure 1b, traffic density increases in the $x$ direction for $t = 0$, a shockwave develops which shows a slope of $\Delta q/\Delta k$. The traffic state on this wave changes discontinuously. Decreasing density in the $x$-direction for $t = 0$ gives rise to a fan of characteristics in which all intermediate densities occur. This shows that the flow from congestion to a downstream free-flowing situation always runs via the congestion regime. The traffic flow leaving congestion is, therefore, optimal in the LWR model.

A number of efforts have been made to refine this LWR model. Higher order models such as those of Payne (3) and Zhang (4) add a second equation that consider the effects of relaxation. By doing so, unstable properties of traffic flow can be modelled.

A second type of extension on the LWR model segregates traffic flow further. The freeway to be modelled can be divided into traffic lanes as was done by Munjal (5) and Daganzo (6); user-classes that have no effect on the flow of traffic can be identified (7); and several vehicle types can be distinguished as in Wong (8). The new behavioural theory of Daganzo (9) also
fits in this category of extensions. This theory combines differences in traffic lanes, vehicle classes and changing driver behaviour.

In this paper the LWR model is extended to include a number of different vehicle classes. As distinct from Wong (8) and Daganzo (9), central to this undertaking is the retention of the numerical scheme for the LWR model described by Daganzo (10) and Lebacque (11). It is also assumed that the vehicles classes can be described by fundamental diagrams similar in shape and that traffic flow speed remains homogeneous for all vehicles.

The basic properties of the LWR model are retained. This enables the study of the intrinsic properties of the model in an analytically constructed solution. Those properties that emerge from the numerical method are dealt with separately.

The introduction of heterogeneous vehicle properties and driver behaviours has already been applied in other types of traffic flow models. The individual treatment of vehicles and drivers give microsimulation models their heterogeneous nature. The mesoscopic continuum models were extended by Helbing (12) and Hoogendoorn (13).

Heterogeneous dynamic traffic models are extremely useful for ITS applications. The introduction of vehicle based ITS measures (e.g. cruise control systems, ...) affects driving behaviour and vehicle properties. Due to the step-wise introduction, ITS leads to a segregation in driving behaviour that can be modelled with a multi-class model. The incorporation of these heterogeneous properties within a macroscopic model leads to a fast tool that can be used in real-time applications.

In the following section we examine the analytical framework in which the heterogeneous flow will be modelled. We then develop a numerical scheme that is based on the existing LWR scheme. Next, we examine an analytical and numerical case study. We end with a number of observations that illustrate the relation with the much used passenger-car equivalents.

**ANALYTIC FRAMEWORK**

In this section we extend the LWR model for a link with constant road characteristics to several classes of vehicles and/or drivers. The rest of the discussion confines itself to two classes only, without loss of generality. The variables that apply to the separate classes are indicated by the indices 1 and 2.

The characteristic property of each class is described by its fundamental diagram. When we confine our focus to vehicles-drivers from one of the two classes on the freeway under consideration only, a fundamental diagram \( q_i = Q^h_i(k_i) \) applies. Superscript \( h \) in the formula indicates the homogeneous flow of vehicles: only vehicles from the class under consideration travel this link.

For the two classes of vehicles we assume fundamental diagrams of similar shape, though they need not share the same capacities and maximal densities. Figure 2 shows two such diagrams for the two vehicles classes. Since the shape of both diagrams is identical, the fundamental diagram for class 2 vehicles can be seen as a scaled version of the class 1 diagram:

\[
Q^h_{z1}(k) = r Q^h_{z2}(k/p)
\]  

(2)
Figure 2: Two similarly shaped fundamental diagrams for two vehicles classes.

The scaling factor $r$ denotes the proportionality between the two diagrams. This also determines the relation between a number of special points:

$$
k_{\text{max}1} = r k_{\text{max}2}
$$

$$
q_{\text{max}1} = r q_{\text{max}2}
$$

$$
k_{j1} = r k_{j2}
$$

$$
u_{j1} = u_{j2}
$$

Knowing the scaling factor $r$, we can describe the homogeneous class 2 flow using the class 1 diagram as follows:

$$
q_2 = \frac{Q_{c1}^h(r k_2)}{r} \quad \text{or} \quad r q_2 = Q_{c1}^h(r k_2)
$$

This last formula leads us to the introduction of a transformation. Consider a new vehicle class $2^*$ that relates to class 2 as follows:

$$
q_2^* = r q_2
$$

$$
k_2^* = r k_2
$$

Speeds remain insensitive to this transformation:

$$
u_2^* = \frac{q_2^*}{k_2^*} = \frac{r q_2}{r k_2} = u_2
$$

The same fundamental diagram of class 1 applies to this new class:

$$
q_2^* = Q_{c1}^h(k_2^*)
$$

The scale-factor $r$ can also be interpreted as a passenger-car equivalent (pce). As is the case with pce factors, a vehicle class is transformed to the properties of another representative class. The value of $r$ and the pce always indicate the relation between the capacity of one class in regard to the reference-class.
As we proceed with the derivation of the model, the heterogeneous model is converted to a model for a traffic flow with a number of classes, each of which corresponds to the same fundamental diagram. When we accept that the mixed traffic flow also adheres to this fundamental diagram, the heterogeneous traffic flow is described as follows:

\[
\frac{\partial k_{tot}^*}{\partial t} + \frac{dQ^h_{tot} (k_{tot}^*)}{dk_{tot}^*} \frac{\partial k_{tot}^*}{\partial x} = 0
\]  

(4)

Here the total transformed density of the mixed traffic flow equals:

\[ k_{tot}^* = k_1 + k_2^* = k_1 + r_k k_2 \]

Because of the same fundamental diagram for both class 1 as the transformed class 2*, the total transformed flow becomes:

\[ q_{tot}^* = Q^h_{el} (k_{tot}^*) \]

The average speed equals:

\[ u_{tot}^* = \frac{q_{tot}^*}{k_{tot}^*} \]

The flow for class 1 and the transformed class 2* can then be calculated as:

\[ q_1 = k_1 u_{tot}^* = \frac{k_1}{k_{tot}^*} Q^h_{el} (k_{tot}^*) \]

\[ q_2^* = k_2^* u_{tot}^* = \frac{k_2^*}{k_{tot}^*} Q^h_{el} (k_{tot}^*) \]

We also assume that the speed of both classes conforms in all cases. An equal speed per class, therefore, also applies to a mixed traffic flow. Thus, the various (transformed) classes behave the same. In this way the contribution of a class can be written both in function of density and flow:

\[ \alpha_1^* = \frac{q_1}{k_{tot}^*} = \frac{k_1}{k_{tot}^*} \]

\[ \alpha_2^* = \frac{q_2^*}{k_{tot}^*} = \frac{k_2^*}{k_{tot}^*} \]

The conservation of vehicles applies to each class. Writing out these relations gives:

\[ \frac{\partial \alpha_i^* k_{tot}^*}{\partial t} + \frac{\partial \alpha_i^* q_{tot}^*}{\partial x} = 0 \]

Further computation leads to:

\[ \alpha_i^* \frac{\partial k_{tot}^*}{\partial t} + k_{tot}^* \frac{\partial \alpha_i^*}{\partial t} + \alpha_i^* \frac{\partial q_{tot}^*}{\partial x} + q_{tot}^* \frac{\partial \alpha_i^*}{\partial x} = 0 \]

Applying the conservation law for the total traffic flow (4) this becomes:

\[ \frac{\partial \alpha_i^*}{\partial t} + u_{tot}^* \frac{\partial \alpha_i^*}{\partial x} = 0 \]

This equation shows that vehicle composition along a trajectory is constant. The composition of the traffic flow, therefore, is unable to change faster than the speed of the vehicles. This amounts to a first in first out (FIFO) rule. Class 1 and the transformed class 2* consequently behave like two classes in a user-class problem. Thus, traffic composition, in transformed variables, does not affect the course of the entire traffic flow.
NUMERICAL SCHEME

In a numerical scheme, the given partial differential equations find a practical application. Starting from initial- and boundary conditions we compute an approximate solution for the heterogeneous LWR model using similar shaped fundamental diagrams. The scheme is based on the Godunov scheme as reinterpreted for the LWR model by Daganzo (10) and Lebacque (11). In doing so, the freeway to be modelled is divided into cells with a length of $\Delta x$ in which the values of the traffic variables are assumed to be homogeneous. The change of these variables is calculated per time interval $\Delta t$. Cell length and time unit are adjusted to each other, in such a way that:

$$\frac{\Delta x}{\Delta t} = u_f$$

The approximating density $K$ of class $i$ in the cell bordered by $x - \Delta x/2$ and $x + \Delta x/2$ is calculated at time $t + \Delta t$ as:

$$K_i(t + \Delta t, x) = K_i(t, x) + \frac{\Delta t}{\Delta x} \left[ G_i(t, x - \frac{1}{2} \Delta x) - G_i(t, x + \frac{1}{2} \Delta x) \right]$$

(5)

$G_i$ represents the flows at the cell-transitions. From time $t$ to $t + \Delta t$ this equals $G_i(t, x - \frac{1}{2} \Delta x)$ along the upstream cell boundary. Taking the given transformation into account, these transition flows per vehicle class are further deduced.

In computing these transitional flows, we look at two cells using a simplified notation as shown in figure 3. The upstream cell is indicated by superscript $U$ and the downstream one by $D$. In the upstream cell the fundamental diagram $Q_{el}^{U}(k)$ applies to the first class while a scaled version of this diagram with scaling factor $r^U$ applies to the second class. The diagram $Q_{el}^{D}(k)$ and the scaling factor $r^D$ apply downstream from the cell transition. This numerical scheme explicitly addresses changing freeway characteristics. Both the shape of the fundamental diagram and the class 2 scale factor in the up- and downstream cells can vary.

<table>
<thead>
<tr>
<th>U</th>
<th>$G_i$</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_i^U$</td>
<td>$K_i^D$</td>
<td></td>
</tr>
<tr>
<td>$r^U$</td>
<td>$r^D$</td>
<td></td>
</tr>
<tr>
<td>$Q_{ei}^{U}(k)$</td>
<td>$Q_{ei}^{D}(k)$</td>
<td></td>
</tr>
<tr>
<td>$S^U$</td>
<td>$R^D$</td>
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Figure 3: Upstream and downstream cell in the numeric scheme

The Godunov scheme calculates the transitional flow as a minimum between the sending flow from the upstream cell and the receiving flow in the downstream cell. This sending flow $S$ reflects the volume of traffic that can leave a cell. This equals the flow in the cell under consideration during free-flow, the capacity value applies during congestion. The receiving flow reflects the flow that can enter a cell. During free-flow the capacity applies here, during congestion it equals the flow in the cell:
\[
S(k) = \begin{cases} 
Q_s(k) & \forall k < k_{\text{max}} \\
q_{\text{max}} & \forall k \geq k_{\text{max}}
\end{cases}
\]
\[
R(k) = \begin{cases} 
q_{\text{max}} & \forall k < k_{\text{max}} \\
Q_s(k) & \forall k \geq k_{\text{max}}
\end{cases}
\]

From this point in the text onward \(S^U\) denotes the sending flow of the fundamental diagram of the upstream cell. \(R^D\) denotes the receiving flow of the downstream cell.

Keeping the transformation of class 2 in mind, the total scaled sending flow of the upstream cell must equal \(S^U_{\text{tot}}(K_1^U + r^U \cdot K_2^U)\). The downstream cell's total scaled receiving flow takes account of the downstream scale factor and equals \(R^D_{\text{tot}}(K_1^D + r^D \cdot K_2^D)\). Since the up- and downstream cells incorporate other scale factors, one can not simply compute total transition flow as a minimum of \(S^U_{\text{tot}}\) and \(R^D_{\text{tot}}\). If we want to compare these flows, the sending flow must be transformed from the upstream to the downstream scale factor. To do this, the sending flow is first divided per vehicle class:

\[
S^U_1 = \frac{K_1^U}{K_1^U + r^U \cdot K_2^U} S^U_{\text{tot}}
\]
\[
S^U_2 = \frac{r^D \cdot K_2^U}{K_1^U + r^U \cdot K_2^U} S^U_{\text{tot}}
\]

The values of these sending flows from the upstream cell, that have however been scaled according to the downstream transformation, are denoted by superscript \(D\) and they are equal to:

\[
S^D_1 = S^U_1 = \frac{K_1^U}{K_1^U + r^U \cdot K_2^U} S^U_{\text{tot}} \quad (6)
\]
\[
S^D_2 = \frac{r^D \cdot S^U_2}{S^U_2} = \frac{r^D \cdot K_2^D}{K_1^U + r^U \cdot K_2^U} S^U_{\text{tot}} \quad (7)
\]

The total downstream scaled sending flow can be compared to the receiving flow \(R^D_{\text{tot}}\) and equals:

\[
S^D_{\text{tot}} = S^D_1 + S^D_2 = \frac{K_1^U + r^D \cdot K_2^U}{K_1^U + r^U \cdot K_2^U} S^U_{\text{tot}} \quad (8)
\]

Since the sending and receiving flows are now known in the same classes, the total transition flow \(G\) in the downward scaled variables can be calculated as:

\[
G^D_{\text{tot}} = \min[S^D_{\text{tot}}, R^D_{\text{tot}}] = \min \left[ \frac{K_1^U + r^D \cdot K_2^U}{K_1^U + r^U \cdot K_2^U} S^U_{\text{tot}}, R^D_{\text{tot}} \right] \quad (9)
\]

This total transition flow can be separated per class according to the contribution of each class in the sending flow. For the moment we stay with the downstream scaling so that they equal:

\[
G^D_i = \frac{S^D_i}{S^D_{\text{tot}}} G^D_{\text{tot}}
\]

Re-scaling to the original classes happens when:
Further computation, by substituting (6), (8) and (9) in (10) and (7), (8) and (9) in (11) gives the general formulation for transition flows per class:

$$G_i = G_i^D = \frac{S_i^D}{S_{tot}^D} G_{tot}^D$$

(10)

$$G_2 = G_2^D = \frac{S_i^D}{r_r^D S_{tot}^D} G_{tot}^D$$

(11)

In combination with basic equation (5) a complete numerical scheme has been formulated that can be used to compute a link's dynamic traffic evolution.

When a vehicle stream is described by several scaled classes, the transition flow per class equals:

$$G_i = \min \left[ \frac{K_i^U}{(K_i^U + r_u^U K_j^U)} S^U \left( \sum_{j=1}^{n} r_j^U \cdot K_j^U \right), \frac{K_i^U}{(K_i^U + r_d^D K_j^U)} R^D \left( \sum_{j=1}^{n} r_j^D \cdot K_j^U \right) \right]$$

(12)

in which the scaling factors for the first reference class equal $r_1^U = r_1^D = 1$.

When the freeway characteristics along a freeway remain unchanged, the numerical scheme for two vehicle classes can be simplified. In this case the up- and downstream scaling factors are equal so that:

$$G_i = \frac{K_i^U}{(K_i^U + r_r^U K_j^U)} \cdot \min[S^U \left( K_1^U + r_2^U K_2^U \right), R^D \left( K_1^D + r_2^D K_2^D \right)] = \frac{K_i^U}{(K_i^U + r_r^U K_j^U)} \cdot G_{tot}^*$$

(13)

The total scaled transition flow is defined by:

$$G_{tot}^* (t) = \min[S^U \left( K_1^U + r_2^U K_2^U \right), R^D \left( K_1^D + r_2^D K_2^D \right)]$$

Dividing this total transition flow over the classes is only carried out in a subsequent step. This enables the inclusion of vehicle delay-periods in the calculation of transition flow per class. Daganzo (7) formulated a rule of delay that takes account of the length of delays in the composition of the transition flow. This rule of delay decreases the dispersion of the numerical scheme when the composition of the traffic flow alters.

To this end, new densities and transition flows are defined as a function of the delay-period.

The amount of traffic per class $i$ in the upstream cell with a delay $\tau$ at time $t$ is written as: $\Delta x_i K_{i\tau}^U (t)$. This gives the number of class $i$ vehicles that entered this cell immediately following on time interval $(t-\tau)$.

Analogue to this, the required transition flows $G_i(t)$ can be divided according to the delay-periods $\tau$ of which the traffic is composed $G_{i\tau}(t)$.

The following updating rules apply to this extended notation:
\[
\Delta x. K^D_{i\Delta t}(t + \Delta t) = \Delta t.G_i(t) = \Delta t. \sum G_{i\tau}(t)
\]

(15)

Traffic that enters a cell receives delay-period \( \Delta t \) at the next time-unit.

\[
K^U_{i(\tau + \Delta t)}(t + \Delta t) = K^U_{i\tau}(t) - \frac{\Delta t}{\Delta x}. G_{i\tau}(t)
\]

(16)

Traffic remaining in the cell increases its delay-period with the time interval \( \Delta t \).

In order to respect the FIFO rule, delays are recorded. The computed total transition flow \( G^*_{\text{tot}}(t) \) is then divided according to class and composed by traffic with the longest delays. Formulating transition flows per class is done as follows:

\[
G_{i\tau}(t) = f_{\tau}. \frac{\Delta x}{\Delta t}. K^U_{i\tau}(t)
\]

Calculating factor \( f_{\tau} \) starts at the longest delay \( \tau_{\text{max}} \) in the upstream cell. Using diminishing delay-periods all factors per cell are subsequently calculated.

When the total transition flow has not yet been achieved, that section of traffic that has experienced the longest delay is sent on to the next cell. Mathematically this means that:

\[
f_{\tau} = 1 \quad \text{so long as} \quad \sum G_{1p}(t) + r. \sum G_{2p}(t) + \frac{\Delta x}{\Delta t}. K^U_{i\tau}(t) \leq G^*_{\text{tot}}
\]

If this equation no longer applies it means that not all of the traffic experiencing this length of delay can be sent on to the next cell. \( f_{\tau} \) is then calculated as follows:

\[
f_{\tau} = \frac{\Delta t}{\Delta x} \left[ \frac{G^*_{\text{tot}} - \left( \sum G_{1p}(t) + r. \sum G_{2p}(t) \right)}{K^U_{i\tau}(t) + r.K^U_{2\tau}(t)} \right]
\]

\( f_{\tau} = 0 \) applies to all traffic experiencing a shorter delay.

In conclusion, this gives:

\[
G_i(t) = \sum G_{i\tau}(t) = \sum f_{\tau}. \frac{\Delta x}{\Delta t}. K^U_{i\tau}(t)
\]

whereby the factor \( f_{\tau} \) per diminishing delay \( \tau \) is calculated as follows:

\[
f_{\tau} = \min \left\{ 1, \max \left[ 0, \frac{\Delta t}{\Delta x} \left[ \frac{G^*_{\text{tot}} - \left( \sum G_{1p}(t) + r. \sum G_{2p}(t) \right)}{K^U_{i\tau}(t) + r.K^U_{2\tau}(t)} \right] \right] \right\}
\]

The transition flows per class achieved are composed, therefore, of the longest waiting traffic from the upstream cell and meet the following criterion:

\[
G_1(t) + r.G_2(t) = G^*_{\text{tot}}(t)
\]
CASE-STUDY

In this section we develop an analytical solution to a freeway traffic problem and also present a numerical analysis.

The case study considers a freeway without access and exit roads. The freeway is divided into three sections. In the first section, where \( x < x_1 \), the fundamental diagram \( Q_{e1}^0(k) \) for class 1 vehicles applies. Class 2 vehicles behave according to a scaled version of this \( q-k \) fundamental diagram. The scaling factor in this first section is \( r^A = 2 \).

In a second section, where \( x_1 < x < x_2 \), the same fundamental diagram \( Q_{e1}^0(k) \) applies to the class 1 vehicles. The scaling factor for the scaled diagram that describes class 2 vehicles, is \( r^B = 4 \).

The third section, where \( x > x_2 \), shares the properties of the first section. The fundamental diagram \( Q_{e1}^0(k) \) applies to class 1 and the scaling factor for class 2 is \( r^A = 2 \).

This theoretical case study can be seen as a freeway carrying private- and goods traffic, and where the freeway inclines sharply at the level of the second section.

An empty freeway is assumed as the initial condition:

\[
k(t, x_0) = 0 \quad \forall x > x_0
\]

The upstream traffic demand starts at \( t_0 \) with an equal inflow of class 1 and class 2 vehicles:

\[
q_1(t_0, x) = \frac{1}{4} q_{max1} \quad \forall t_0 < t < t_1
\]

At time \( t_1 \) traffic composition changes to a homogeneous class 1 traffic flow.

\[
q_1(t_0, x) = \frac{1}{2} q_{max1} \quad \forall t_1 \leq t
\]

\[
q_2(t_0, x) = 0
\]

Total traffic demand, therefore, does not change at \( t_1 \), only the composition of the traffic changes.

Figure 4 depicts the fundamental diagram, figure 5 gives the analytic computation of this problem. The characteristics, the constant flow solution lines, are indicated by a thin line, shock waves by a bold line.

The construction of the solution starts from the origin with a fan between the empty traffic state and the state where \( q^*_{tot} = q_1 + r^A q_2 = \frac{3}{4} q_{max1} \). This fan is followed by kinematic waves that arise from the boundary condition and travel to \( t_1 \). A segregation trajectory beginning at \( t_1 \) indicates the end of the class 2 vehicles. A shock wave starting at the same point reflects the break with the new traffic demand: \( q^*_{tot} = q_1 = \frac{1}{2} q_{max1} \). Although total traffic demand remains the same, a shock wave does occur. This happens, of course, because the total scaled traffic demand actually does differ.
Traffic demand in the first period $0 < t < t_1$, is too large to be processed by the second section. This is due to the fact that the transformed total flow with scaling factor $r^B$ is $q^*_t = q_1 + r^B q_2 = \frac{5}{4} q_{max1}$. This causes a regressive shock wave during the opening fan in the
first section. Those characteristics that formerly flowed into the second section, show smaller slopes in this second section.

The congestion zone in the first section grows up to the shock wave of the changing traffic demand. At this point a forward shock wave diminishes the length of the congestion. When the segregating trajectory reaches the second section, the upstream bottleneck for section 1 disappears. This gives rise to a fan that dissolves the congestion in the first section thus increasing the speed of the closing shock wave.

The trajectory that marks the changing composition of traffic demand causes a new fan when it arrives in the third section. Before this trajectory reaches the third section, the speed of the characteristics increases at the border of section 2 and 3. Because the class 2 vehicles in the traffic flow suddenly assume a smaller scale factor, it is as if the capacity of the total traffic flow increases at that point. Subsequent to this segregating trajectory $q_2$ equals zero. This is why the same fundamental diagram applies to section 2 and 3 from that point.

Figure 6 shows the $t$-$x$ diagram of the numerical simulation. The figure gives the total flow $q_{tot}$ ($= q_1 + q_2$) using a colour scale. Black equals zero flow, white indicates capacity flow $q_{max1}$. In this numerical method the $t$-$x$ solution space is sub-divided into cells. For each time interval $\Delta t$, density according to (5) was computed for each cell with a length of $\Delta x$. When computing transition flows, the delay rule was applied for constant road properties. This dispersion correction was not used for the section transitions at $x_1$ and $x_2$.

![Figure 6: Numerical solution of the case study in the $t$-$x$ diagram.](image)

The similarities between the numerical calculation in figure 6 and the analytic solution in figure 5 are clearly visible.
DISCUSSION

This section takes a closer look at the significance of the scale factor in the presented heterogeneous model. The flow of vehicles was divided into classes. All vehicles inside a class are assumed to share the same properties. Scale factor \( r \) denotes how vehicles from various classes differ. It corresponds to the relation between the 'jam' densities and the capacities. The space occupied by a vehicle of a particular class at speed \( u \) is indicated by \( s_i(u) \). Since the spacing is inverse to density, the proportionality factor between the average spaces is also related to the scale factor.

\[
\frac{r}{k_{\text{max}2}} = \frac{k_{i1}}{k_{j2}} = \frac{s_j(u)}{s_i(u)}
\]

Using scale factors to divide traffic flow into various classes enables us to highlight the following heterogeneous characteristics:

- Differences in vehicle characteristics, e.g. the physical length of vehicles
- Differences in driver characteristics, e.g. calm and aggressive drivers
- Differences in road characteristics that affect the capacity of particular types of vehicles, e.g. tunnel.

Conversely, this extended LWR model cannot describe the following characteristics:

- Different maximum speed limits. This is relevant both in terms of vehicle characteristics (e.g. heavy vehicles versus private cars), road characteristics (e.g. slope) and driver characteristics (e.g. fast drivers)
- Differences in speed. A homogeneous traffic speed is assumed. This is why overtaking and multilane highways are insufficiently modelled.
- Differences in acceleration and reaction time: In the LWR model, traffic is assumed to agree always with the fundamental equilibrium diagram. The reaction time of drivers is neglected and the modelling of acceleration behaviour can be improved.

If we take passenger-cars as a class of reference, then the scaling factor equals the generally used passenger-car equivalents (pce). As in the Highway Capacity Manual 2000, pce includes differences in vehicle characteristics (e.g. freight), driver characteristics (e.g. commuters vs. recreational) and road characteristics (e.g. slope, multilane, ..). To do this, an equivalent is determined at an hourly basis and this also takes account of differences in (maximum) speed, acceleration and reaction time. Averaging the heterogeneous effects on an hourly basis, enables the inclusion of differences that could never otherwise be described dynamically by that kind of factor. Using the known pce values in the heterogeneous LWR model gives us a readily applicable heterogeneous LWR model that can also indirectly address speed differences.

Simplified versions of this heterogeneous model lead to the classical LWR model. When traffic composition on a link does not change during the model period, a weighted fundamental diagram can be used. The homogeneous LWR model also applies when the scaling factor \( r \) is identical for all modelled links. Scaling the traffic demand to the reference class satisfies in the latter case. The added value of this model lies within changing vehicle compositions during the modelled period in combination with varying pce over the modelled network.

Further research is planned to present speed differences between classes in a heterogeneous model with greater accuracy. The requirement of using similarly shaped fundamental diagrams will then also disappear.
Homogeneous speed will continue to apply when both vehicle classes experience congestion. Free-flow regimes will allow for different speeds per class. Because the speed differences per class are related to the possibilities of overtaking, extra lane information will also be required.

CONCLUSIONS

This paper extended the LWR model to heterogeneous traffic, based on the following assumptions:

• Vehicle-flow was divided into classes. When vehicles of one class exclusively occupy a freeway, a homogeneous fundamental diagram applies.
• The fundamental diagrams for all classes are similar in shape. This limits the applicability of this method to a certain extent. It imposes a uniform maximum speed for the various classes. But it also gives the opportunity to scale the various vehicles-classes against a reference class. These scaling factors can differ per link and correspond with the known passenger-car equivalents.
• Speed in the mixed traffic flow is homogeneous over all classes. Interactions between the various vehicles do not affect the fundamental diagrams of the various classes. The basic properties of the LWR model remain valid and the usual numerical scheme remains applicable. This results in an readily applicable extension of the LWR model to describe a heterogeneous traffic flow using known pce factors.

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REFERENCES


